

Hydrodynamic Modeling of Sailing Yachts

Stefan Harries, Technical University of Berlin **Claus Abt,** Technical University of Berlin **Karsten Hochkirch,** Friendship Systems, Berlin

ABSTRACT

In modern yacht design geometric modeling is regarded to be directly related to the hydrodynamic performance of the shape of the hull and its appending elements – usually the keel, often with winglets, and the rudder. While the traditional way of shape design – i.e., drawing, model building, tank testing, modifying ... – is both time consuming and expensive, a complementing approach shall be discussed within this paper. The approach is called hydrodynamic modeling since it tightly combines the hydrodynamic analysis and the geometric modeling in the design process. It is based on advanced Computational Fluid Dynamics (CFD) methods for flow field analysis and unique parametric modeling techniques for shape generation.

The geometry of a yacht is entirely described via important form parameters as discussed in detail by the authors at the 1999 CSYS. The canoe body of the yacht is modeled from a small set of longitudinal curves which provide all parameters needed for sectional design. The longitudinal curves themselves being created via form parameters, a fully parametric description of the hull is achieved which allows to create and modify the geometry in a highly sophisticated manner. The fairness of the shapes is an intrinsic part of the form generation procedure. Apart from the canoe body the keel represents the most pronounced hydrodynamic design element, dominating lift and righting moment of a yacht but also causing a non-negligible resistance component called induced drag. Keel, bulb and winglets are also specified in terms of form parameters.

An application of hydrodynamic modeling is given for an IACC-yacht. Formal optimization can be successfully employed to identify improved and, eventually, optimal configurations. A reasonably small set of parameters (free variables) was selected and systematically varied making use of a fully-automatic optimization scheme. Two optimization examples are presented in order to demonstrate the potential of the approach: (a) the optimization of a keel-bulb-winglet configuration so as to find a minimum drag solution for a given sideforce and (b) the optimization of the bare hull with respect to wave resistance.

The examples can be regarded as representative for both racing and touring yachts with draft restrictions and illustrate the methodology of hydrodynamic modeling.

NOMENCLATURE

B_{max}	maximum beam at deck level
C_B	block coefficient
C_P	prismatic coefficient
E_2	fairness criteria
F_n	Froude number
IACC	International America's Cup Class
L_{PP}	length between perpendiculars
VPP	Velocity Prediction Program
$x_{B_{max}}$	longitude of maximum beam
х _{CB}	longitudinal center of buoyancy
T _{max}	maximum draft

INTRODUCTION

Designing a yacht, in particular its hull geometry and appendages, is a process of creativity, skill, experience and art – independent of whether the naval architect chooses to express his or her ideas by means of a traditionally drawn lines plan or whether the designer decides to apply a computer aided design (CAD) system to create a product model. Full benefit can be gained from the latter when an integrated process of modeling and analysis is established in which design variations can be evoked and assessed efficiently.

In geometric modeling and particularly in yacht design many CAD systems are now built on an outstanding mathematical curve and surface representation technique known as B-splines. Originating in free-form design, the underlying methodology of most of these systems is the interactive shape generation where points – e.g. the vertices of the B-spline's defining polygon or polyhedron – are positioned in three-dimensional space. Achieving the desired form generally is a laborious undertaking since the results need to be suitably fair while specific constraints have to be taken into account, e.g. the displacement or the length of a water line important to the rating rules under consideration. Then the process of manual vertex manipulation becomes rather tedious and systematic modifications in order to improve the shapes with regard to their hydrodynamics become inapt.

Instead of interactively handling the lowest entities of the underlying mathematical model (i.e., the vertices) a different approach has been pursued which is aimed at expressing the geometry in terms of high level descriptors for the intended shapes (i.e., form parameters).

Following the stage of shape creation, the design may be analyzed for its various characteristics. The hydrodynamics being of supreme importance to racing yachts, a state-of-the-art system of Computational Fluid Dynamics can be used to examine the performance. Modern flow codes have reached the maturity to rank different designs in respect to resistance and lift (i.e., side force). A potential flow code with good response time was therefore applied to numerically analyze the flow about the hull and a keel-bulb-winglet configuration.

The potential of linking the two stages of geometric modeling and hydrodynamic analysis tightly together, becomes apparent when utilizing an integrated environment in which modeling, analysis, evaluation and modification can be repeated systematically within short turn-around time.

Within this paper the parametric approach will be discussed and examples will be shown for an IACC yacht, see section on geometric modeling. The hydrodynamic optimization of an IACC canoe body and its keelbulb-winglet configuration by means of a formal strategy and a fully-automatic process will be presented, see section on optimization. A full optimization in the light of a velocity prediction program (VPP) is discussed.

GEOMETRIC MODELING

In parametric modeling design ideas are usually expressed by descriptors that imply higher level information about the object to be created. Often, relationships and possible dependencies between entities are considered. In addition, the descriptors may also represent complex features that the product is to assume. When modeling geometry, the descriptors are called form parameters, three types of which can be distinguished:

Differential form parameters like tangents and curva-

tures (e.g. angle of entrance of the design waterline),

- **Positional** form parameters like points to be interpolated (e.g. breadth of the waterline at the transom),
- **Integral** form parameters like area, volume and centroid information (e.g. center of flotation).

Well-defined parameters facilitate the modeling process since the designer can focus his or her attention on the outcome rather than on the input, assuming that the form generation procedure automatically brings about the specified geometry by itself.

In the subsections to come, first the parametric modeling of (bare) hulls shall be briefly reviewed as introduced by HARRIES AND ABT (1999b) and HARRIES (1998). New features will be presented so as to cope with additional constraints originating from class rules. Following this, the parametric modeling of appendages shall be outlined as needed for hydrodynamic optimization.

Hull

In the novel parametric approach to the design of sailing yachts by HARRIES AND ABT (1999b) the process of modeling surfaces of complex geometry is based on laying out a set of cross-sectional curves and, subsequently, generating a surface by means of lofting (LETCHER, 1981) or skinning (WOODWARD, 1986, 1988). Following the classic naval architect's technique of describing a ship's geometry in terms of longitudinal curves (i.e., basic curves) from which design sections are derived, the design sections (i.e., the cross-sectional curves) define the interpolating surfaces and determine the shape of the envisioned hull.

A new system called *FRIENDSHIP Modeler*,¹ has been introduced which is based completely on parametric design principles. One of the key features of the

¹Form parameter oRIENteD SHIP Modeler http://www.friendship-systems.com



FIGURE 1: Levels of the hydrodynamic modeling system

Layer		Objective function	Constraints	Design/Free variable
2	Hydrodynamic	Performance	Feasible domains of the	Properties defined by the
	optimization		design variables	constraints of layer 1 e.g.
				C_P, x_{CB}
1	Parametric hull	Fairness of basic curves	Displacement, x_{CB} ,	Direct geometrical
	generation with implicit	and sectional fairness	interpolation of the deck,	properties of the basic
	measurement constraints		design waterline,	curves e.g. B_{max} , T_{max} ,
	and physical properties		centerplane, tangents,	$L_{PP}, x_{B_{max}}, x_{T_{max}}, B_{stern},$
	from form parameter		tangent plane, shape	B _{bow}
	defined basic curves		modification curves,	
			measurement marks	
0	Parametric B-spline	E ₂ Fairness	Interpolation, enclosed	Vertex coordinates,
	generation		area, centroid position,	vector sizes
			tangential properties,	
			curvature	

TABLE 1: Direct parametric modeling

Layer		Objective function	Constraints	Design/Free variable
3	Hydrodynamic optimization	Performance	Feasible domains of the design variables	Properties defined by the constraints of layer 2 (e.g. C_P , x_{CB}) and free variables of layer 1
2	Geometric optimization	Global and local fairness of the hull	Displacement, formula constraints, form parameters e.g. C_P , C_B , x_{CB} , lateral area, waterplane area, center of flotation, convexity	A subset of the free variables from layer 1
1	Parametric hull generation with implicit measurement constraints and physical properties from form parameter defined basic curves	Fairness of basic curves and sectional fairness	Displacement, <i>x_{CB}</i> , Interpolation of the deck, design waterline, centerplane, tangents, tangent plane, shape modification curves, measurement mark	Direct geometrical properties of the basic curves e.g. B_{max} , T_{max} , L_{PP} , $x_{B_{max}}$, $x_{T_{max}}$, B_{stern} , B_{bow}
0	Parametric B-spline generation	E_2 Fairness	Interpolation, enclosed area, centroid position, tangential properties, curvature	Vertex coordinates, vector sizes

TABLE 2: Advanced parametric modeling

approach is the generation of B-spline curves and surfaces by means of variational calculus. Instead of interactively manipulating the B-spline's control points, the (free) vertices are computed from a geometric optimization which employs fairness criteria as measures of merit and captures the specified form parameters as equality constraints.

Modeling a hull thus becomes the task of selecting the form parameters to be taken into account and assigning suitable values to them. This can be done by evaluating an existing design and remodeling it or, alternatively, specifying a set of form parameters from scratch. As soon as an initial shape is produced changes can be systematically brought about by varying one or several parameters.

Form parameters can be individually addressed and changed. Nevertheless, the interdependency of form parameters needs to be considered. For instance, pushing the center of flotation aft while pulling the center of buoyancy forward can only be accommodated within subtle limits unless non-yacht like shapes are intended.

Within hydrodynamic optimization, the direct use of physical properties like displacement and center of buoyancy can be successfully employed, see HARRIES AND ABT (1999a). Variations can be evoked efficiently but the initial guess should be reasonably close to where



FIGURE 2: Parametrically designed IACC yacht with circular sections



FIGURE 3: Parametrically designed IACC yacht with trapezoidal sections



FIGURE 4: IACC canoe body designed by parametric modeling



FIGURE 5: Perspective view of an IACC canoe body designed by parametric modeling

Kiwihunter.iac

name	Kiwihunter	
// parameters of layer MeterClass S DSP lcBuoy lcFlot lcLatArea	2 (optional) 24.0 280.0 19.5 10.50 11.00 10.00	<pre>// measurement value // sail area // displacement // longitudinal center of boyancy // center of flotation // center of lateral area canoe body</pre>
<pre>// parameters of layer fairFlag nosec noVerticesPerSection useFlatInterpol atCurveParameter</pre>	1 (required) 0 11 8 0 0.2	<pre>// fair skinning switch // number of sections // vertices per section // intial velocity of sections switch // parameter of flat interpolation</pre>
<pre>// keel contour design_elevation design_length design_draft design_draft_x</pre>	0.2 20.0 0.76 9.0	<pre>// IACC - measurement level // lenght at measurement level // maximum draft of canoe body // position of max. draft</pre>
incline_bow incline_stern	10.0 5.0	// stem contour modifier // stern contour modifier
overhang_bow overhang_stern	1.5 2.0	// overhang at bow // overhang at stern
design_freeboard	1.2	<pre>// constant freeboard (for simplicity)</pre>
<pre>// deck beam_bow maxbeam maxbeam_x beam_stern angle_bow angle_stern</pre>	0.3 4.8 11.9 2.8 14.0 13.0	<pre>// beam at bow // maximum beam // position of maximum beam // beam at stern // angle of waterlines at deck // angle of waterlines at stern</pre>
<pre>// flare at deck deck_flare_bow flare_change_bow deck_flare_max_beam flare_change_stern deck_flare_stern</pre>	8.0 0.0 30.0 20.0	// at stem // gradient at stem // at maximum beam // gradient at stern // at stern
<pre>// deadrise deadrise_bow deadrise_change_bow deadrise_max_draft deadrise_change_stern deadrise_stern</pre>	0.0 0.0 0.0 0.0 0.0 0.0	// // // //
<pre>// flat of side flat_bow flat_change_bow flat_max_beam flat_change_stern flat_stern</pre>	0.25 3.0 0.5 -5.0 0.28	<pre>// value at stem // gradient at stem // value at max. beam // gradient at stern // value at stern</pre>
// actual waterline dwl_max_beam dwl_max_beam_x dwl_tangent_bow_dist	3.8 10.6 0.1	// maximum beam // position of max. beam // waterline entry modifier

TABLE 3: Set of parameters describing completely the generated hull surface

the optimal shape is expected. Sometimes, however, it is beneficial that an initial design is changed considerably and more than initially assumed. (This might be due to the lack of experience when developing an entirely new project.) Consequently, giving the modeling process more freedom will greatly assist in finding an optimum – which might then even lie outside the naval architect's conventional experience.

Furthermore, the resulting shapes may sometimes not suit the requirements imposed by class rules. Typically, when optimizing for wave resistance on a downwind course at medium Froude numbers bulbous-like sections in the vicinity of the bow appear, contradicting convexity constraints. (This is due to a favorable redistribution of displacement – at least for heavy displacement yachts where dynamic lift does not play an important role.) Considering class rules at the early stage of shape generation already establishes a tangible advantage.

A special branch of the *FRIENDSHIP Modeler* – called *IACC-Friend*² – has been derived by the authors extending the direct modeling approach to comply with the IAC-Class rules (IACC, 1997).

In order to meet the hull's convexity requirement an additional optimization layer has been introduced in the design process. Tab. 1 shows the three layers which are used in the direct design mode where the sectional area curve forms an integral part of the input to the modeling, see also HARRIES AND ABT (1999b). Tab. 2 presents an advanced approach where large shape variations can be realized while class requirements are simultaneously fulfilled. An additional layer is introduced to balance the shape changes due to a hydrodynamic optimization with the restrictions given by the rule.

For instance, the longitudinal position of the maximum breadth of the design waterline is a typical and effective design variable for a hydrodynamic optimization. The longitude of the maximum beam of the deck should then be utilized as a free variable at a level where formula constraints are accommodated – i.e., at layer 2 in Tab. 2 – such that the hull maintains its convexity. At this level typical integral form parameters which are selected as design variables are implemented as equality constraints. Positional and differential form parameters are generally treated at layer 1 but may be passed to either level 2 or 3 depending on the design problem at hand. At the lowest level – i.e., at layer 0 – the B-splines are computed according to the input received from levels 1 to 3.

Because of the underlying fairness criteria employed to determine the B-spline curves, the shapes created from the parametric approach are intrinsically fair. The fairness criteria (see HARRIES AND ABT, 1999b) are minimized in an optimization, favoring circular sections – i.e., *natural* shapes of roundish character. For better shape control an additional parameter has been devised which allows to conveniently design sections with straight or straightened parts typical of trapezoidal sections. This new form parameter is the vertical position which a transverse B-spline curve has to interpolate at a predefined curve parameter.

The definition of the form parameters is shown in Tab. 3.

Appendages

Similar to the direct design mode for modeling the canoe body of a yacht, the keel fin and bulb can be parametrically described and modeled. While the fin and winglet may be described readily from excellent wing section data – for instance via scaling, blending and merging in a longitudinal sweep operation – the bulb generally is a free-form object. A bulb's volume and its distribution for example should be accurately defined by means of a sectional area curve, enabling the designer to specify the mass and center of gravity.

An advanced feature of this approach is the method of modifying natural shapes, i.e., shapes that originate from the B-spline optimization when neglecting centroid information. The centroid of the section is modified relative to an unconstrained optimization. This means: After computing a section the highest and lowest centroid position is determined from a vertex transformation in which a zero curvature condition is applied at the upper and lower ends of each B-spline curve, respectively. From this the extreme centroid locations are calculated and mapped onto an unified parameter space. Subsequently, each design section is computed anew with the desired centroid location retrieved from the basic curve which defines the centroid modification along the bulb axis. The basic curves defining the contour, sectional area curve and the centroid modifier are



FIGURE 6: Basic curves of an IACC keel bulb

²International Americas Cup Class Form parameter oRIENteD modeler



FIGURE 7: IACC keel bulb with natural vertical centroid (neutral centroid modifier)



FIGURE 8: IACC keel bulb with modified centroid – low to neutral



FIGURE 9: IACC keel bulb with modified centroid – high via low to neutral

displayed in Fig. 6 for an example bulb. Naturally, those basic curves are also determined via form parameters. The bulbs depicted in Fig. 7 to Fig. 9 exactly feature the same weight and longitudinal center of gravity but originate from changes in the centroid modifier.

For a hydrodynamic optimization the form parameters of all four basic curves can be readily applied. The volume and center of gravity of the bulb are usually to be kept constant while the tangents of the sectional area



FIGURE 10: Optimization Process

curve may be varied. Also, the contours are subject to possible change. Of course, from the set of bulb form parameters any suitable subset can be selected.

OPTIMIZATION

In the proposed parametric design method a yacht's geometry is created in terms of its *direct properties* as expressed by its form parameters. The hull is determined from optimizing the fairness criteria and, consequently, the generated shapes

- · accurately meet all desired properties and
- intrinsically acquire excellent fairness.

Both features are key prerequisites for the optimization of a yacht's most important *indirect properties*, i.e., its various hydrodynamic qualities like resistance, seakeeping and lift-drag ratio.

Figure 10 displays the synthesis model for the formal hydrodynamic optimization of yachts. The synthesis model comprises four stages:

- Model of form generation: Parametric design via the *IACC-Friend* Modeler.
- **Model of hydrodynamic analysis:** CFD simulation by means of a state-of-the-art flow code.

- **Model of design evaluation:** Integral lift and drag forces.
- Model of optimization: Non-linear programming techniques.

Optimizations take place at two levels. In an outer loop important form parameters are systematically varied so as to improve a suitable hydrodynamics criterion, e.g. the drag for a given sideforce at a predefined boat speed. In an inner loop, the geometry is optimized as discussed in the previous subsections.

The optimization scheme in the outer loop is based on a conjugate gradient method as described by PRESS ET AL. (1988). The algorithm compromises two steps which are alternately repeated until convergence. In the first step, the gradient of the measure of merit is computed with respect to the free variables at a base point. In the second step, a promising search direction is identified and a one-dimensional optimization is undertaken, setting out from the base point into the direction of improvement. Here, the Golden Section search method is employed.

The optimum point found along the search line is then used as a new base point and the procedure starts anew: the current gradient is computed and a new search direction is determined for the next line search. Instead of simply using the gradient at the current base point – as it would be done in the method of steepest descent – the new direction is computed from the current and preceeding gradients, promising improved performance in long and narrow valleys of the search space.

In order to demonstrate the feasibility of this approach two cases have been studied:

- A simplified keel-bulb-winglet configuration where the position as well as the sweep and dihedral angles of the winglets have been varied in order to find the configuration with the minimum induced drag for a given sideforce.
- The canoe body where various form parameters have been modified so as to identify the minimum wave resistance of the yacht sailing upright at a moderate Froude number.

The potential flow module of the CFD system SHIPFLOW by LARSSON (1997) was used for the numerical simulations. A reliability study was undertaken for sailing yachts previous to the CFD based optimizations, see PILLER (2000).

Induced Resistance

A topological model was created to set up a keel-bulbwinglet configuration as shown in Fig. 11 which provides a finite set of design variables. For simplicity the



FIGURE 11: Keel-bulb-winglet configuration used for analysis of induced drag (neglecting keel flap)

fin and winglets were generated from NACA 631-012 sections (ABBOTT AND VON DOENHOFF, 1958) while the bulb was modeled as described above. Focusing on induced drag a double body model was considered to be sufficient so as to reduce the computational effort.

A similar configuration was presented by LARSSON (1999), demonstrating that the SHIPFLOW system can be successfully applied to the hydrodynamic design of a keel-bulb-winglet configuration.

Keel, bulb and winglets were discretized using 400, 2000 and 260 panels, respectively, the source strength for each panel being defined by higher order distribution. In order to fulfil the Kutta-Jukowski condition additional strip groups were added at the trailing edges of keel and winglets. Extra strips were introduced to move the tip-vortex to the center of the bulb. For details on the method see JANSON (1997). Fig. 13 displays a typical result of these calculations showing velocity vectors and pressure contours.

Calculations have been carried out for three different leeway angles $(0.0^{\circ}, 2.5^{\circ} \text{ and } 5.0^{\circ})$ and a constant heel angle of 22.0°. The value of induced drag and lift was derived from Treffetz plane analysis. The induced drag associated with an exemplary sideforce of 25 kN was computed via a non-linear interpolation of the results for the different leeway angles.

Only the longitudinal position of the winglets and their dihedral and sweep angle were chosen as free variables within the optimization run (see Fig. 11) to keep the number of computations reasonable. Fig. 12 shows the history of the optimization process: within 10 evaluations the induced resistance could be reduced by approximately 2% – the main change steming from the longitudinal position of the winglets.



FIGURE 12: Optimization history for the induced resistance of a keel-bulb-winglet configuration



FIGURE 13: Perspective view of a simplified keel-bulbwinglet configuration featuring velocity vectors and pressure contours from a potential flow calculation

Wave Resistance

A second set of calculations was carried out for the bare hull of an IACC yacht sailing with neither heel nor leeway. Tab. 3 shows the input file for the parametric modeler *IACC-Friend* which completely describes the generated hull as depicted in Fig. 5.

The wave resistance is calculated by SHIPFLOW, including the non-linear boundary condition at free surface (Fig. 14). For an initial solution approximately 10 iterations were needed. The modified configurations were restarted from a previous solution and converged within a few iterations.

The wave resistance being calculated from pressure



FIGURE 14: Wave contours from a non-linear potential flow calculation for an IACC yacht



FIGURE 15: Optimization history for the canoe body without heel and leeway at $F_n = 0.307$

integration was regarded as a sensible measure of merit. The calculation were carried out for a Froude number of $F_n = 0.307$.

The optimization history is plotted in Fig. 15 for a variation of four free variables:

- longitudinal position of maximum breadth of the design waterline
- bow beam
- · longitudinal position of maximum draft
- angle of the deck line at the stern

The wave resistance of the initial configuration is computed to be 564 N. A tangible reduction of more than 25% is then accomplished within the first few calculations. Within the following iterations improvements in wave resistance can be found as well, however, within the range of only a few percent.

Further variations have been undertaken for different starting values of the free variables. These optimizations resulted in similar trends.

Naturally, it should be kept in mind that the achievable improvement always depends on the quality of the initial system. The better the original design the less potential for changes. Improving a good design is thus much more challenging than gaining on a less sophisticated initial shape. Finally, it needs be pointed out that the accomplished improvements can only be as good as the computational model employed to analyze the flow field. One should therefore not expect the full advantage predicted and experimental validation studies are required to prove the validity of the hydrodynamic optimization based on CFD calculations.

Yacht Optimization – Outlook

The performance of a yacht's hull and its appendages can only be fully appreciated when undertaking a complete velocity prediction (e.g. HOCHKIRCH, 2000), balancing the hydrodynamic and aerodynamic forces and moments for the entire spectrum of courses and wind velocities, see Fig. 16. Furthermore, the optimization of a yacht under race conditions implies many more computations and scenarios than shown here. For instance seakeeping and manoeuvering need to be considered, too. Nevertheless, it has not been an attempt of this paper to tackle this formidable task but to highlight techniques that greatly help in the design process and, eventually, facilitate a more comprehensive optimization.

For an IACC yacht the design evaluation becomes a challenging task in itself since a (probabilistic) measure of merit ought to be considered. For given wind and sea conditions the race time has to be integrated from a VPP yielding the amount of seconds needed for the



FIGURE 16: Complete optimization process for racing yachts

course. The likelyhood of a specific match race scenario in comparison to others will have to be taken into account. However, an extraordinary amount of computations will be required to determine this ultimate measure of merit and to optimize for it.

CONCLUSIONS

The design philosophy of hydrodynamic modeling has been presented. The approach is based on closely relating

- · sophisticated geometric modeling techniques and
- advanced numerical flow field analysis.

A synthesis model for hydrodynamic optimization was applied which comprises the four stages of form generation, fluid dynamic analysis, design evaluation and formal optimization.

Unique parametric modeling techniques have been discussed. Particular emphasis has been given to the

fair design of the canoe body of sailing yachts. An advanced parametric description of both the hull and its appendages is regarded as the key to successful formal optimization. Fully-automatic optimization allows to study a variety of shapes and, eventually, to improve the design with respect to selected measures of merit.

The parametric design of an IACC yacht as well as exemplary optimizations of the yacht's bare hull and keel-bulb-winglet configuration have been shown. The parametrically designed shapes feature excellent fairness. Shape control is accomplished by means flexible sets of parameters (the high level descriptors of the design ideas). Promising improvements could be achieved, demonstrating the potential of hydrodynamic modeling of sailing yachts.

It must be emphasized that the hydrodynamic modeling approach presented cannot replace the experience and skill of the yacht designer. The naval architect still is the key figure to decide which parameters to choose and vary and to assess the validity of the final design. Nevertheless, the approach enables to concentrate on the design task instead of the modeling problem.

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